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# RELATIVE HOMOLOGICAL CATEGORIES

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*Abstract*

We introduce relative homological and weakly homological categories $(\mathbf{C}, \mathbf{E})$, where "relative" refers to a distinguished class $\mathbf{E}$ of normal epimorphisms in $\mathbf{C}$. It is a generalization of homological categories, but also protomodular categories can be regarded as examples. We indicate that the relative versions of various homological lemmas can be proved in a relative homological category.

## 1. Introduction

F. Borceux and D. Bourn [**2**], call a category $\mathbf{C}$ *homological* if it is pointed, regular, and protomodular (in the sense of Bourn [**4**]); in fact they claim that such categories provide the most convenient setting for non-abelian versions of various *standard homological lemmas*, such as snake lemma, 3×3-lemma, etc. Taking this viewpoint, one could still try, however, to introduce a more general setting involving a distinguished class $\mathbf{E}$ of regular epimorphisms, where the homological lemmas are expected to hold only for short exact sequences $K \to A \to B$ with $A \to B$ in $\mathbf{E}$. In particular, there is no reason to exclude the trivial case, where $\mathbf{C}$ is an arbitrary category (say, pointed and with finite limits and finite colimits) and $\mathbf{E}$ the class of all isomorphisms in $\mathbf{C}$. This idea goes back to N. Yoneda [**12**], whose *quasi-abelian* categories can in fact be defined as pairs $(\mathbf{C}, \mathbf{E})$, where $\mathbf{C}$ is an additive category in which the short exact sequences $K \to A \to B$ with $A \to B$ in $\mathbf{E}$ have the same properties as all short exact sequences in an abelian category.

The purpose of this paper is to present a new notion of relative homological and relative weakly homological categories $(\mathbf{C}, \mathbf{E})$, such that whenever $\mathbf{C}$ is a pointed category with finite limits and cokernels/coequalizers, we have:

- $(\mathbf{C}, \text{Isomorphisms in } \mathbf{C})$ always is a relative homological category;
- $(\mathbf{C}, \text{Split epimorphisms in } \mathbf{C})$ is a relative weakly homological category if and only if $\mathbf{C}$ is a protomodular category;
- $(\mathbf{C}, \text{Regular epimorphisms in } \mathbf{C})$ is a relative homological category if and only if it is a relative weakly homological category and if and only if $\mathbf{C}$ is a homological category;

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- $(\mathbf{C}, \text{All morphisms in } \mathbf{C})$ is a relative homological category if and only if $\mathbf{C}$ is a trivial category;
- suitable reformulations of various homological lemmas relative to $\mathbf{E}$ hold in $\mathbf{C}$.

## 2. Axioms for relative homological categories

Throughout the paper we assume that $\mathbf{C}$ is a pointed category with finite limits and cokernels, and $\mathbf{E}$ is a class of morphisms in $\mathbf{C}$ containing all isomorphisms.

**Definition 2.1.** *A pair $(\mathbf{C}, \mathbf{E})$ is said to be a relative homological category, if it satisfies the following conditions:*

*(a) The class $\mathbf{E}$ is pullback stable;*

*(b) Every morphism in $\mathbf{E}$ is a normal epimorphism;*

*(c) $\mathbf{E}$-short-five lemma holds, i.e. in every commutative diagram of the form*

$$\begin{tikzcd} K \arrow[r, "k"] \arrow[d, equal] & A \arrow[r, "f"] \arrow[d, "w"'] & B \arrow[d, equal] \\ K \arrow[r, "k'"'] & A' \arrow[r, "f'"'] & B \end{tikzcd}$$

*with $k = \ker(f)$, $k' = \ker(f')$, and with $f$ and $f'$ in $\mathbf{E}$, the morphism $w$ is an isomorphism;*

*(d) The class $\mathbf{E}$ is closed under composition;*

*(e) If $f \in \mathbf{E}$ and $gf \in \mathbf{E}$, then $g \in \mathbf{E}$;*

*(f) If a morphism $f$ in $\mathbf{C}$ factors as $f = em$, in which $e \in \mathbf{E}$ and $m$ is a monomorphism, then there exists a monomorphism $m'$ in $\mathbf{C}$ and a morphism $e'$ in $\mathbf{E}$, such that $f = m'e'$;*

*(g) If in a commutative diagram*

$$\begin{tikzcd} K \arrow[r, "k"] \arrow[d, "u"'] & A \arrow[r, "f"] \arrow[d, "w"] & B \arrow[d, equal] \\ K' \arrow[r, "k'"'] & A' \arrow[r, "f'"'] & B \end{tikzcd}$$

*$k = \ker(f)$, $k' = \ker(f')$, and morphisms $f$, $f'$, and $u$ are in $\mathbf{E}$, then $w$ also is in $\mathbf{E}$.*

*We will also say that $(\mathbf{C}, \mathbf{E})$ is a relative weakly homological category whenever it satisfies conditions (a)-(e).*

Note that condition 2.1(c) in fact follows from condition 2.1(g). Indeed: under the assumptions of 2.1(c), the morphism $w : A \to A'$ is in $\mathbf{E}$ since $\mathbf{E}$ contains all isomorphisms and condition 2.1(g) holds. Also, it is a well known fact that in this situation $\ker(w) = 0$. Since every morphism in $\mathbf{E}$ is a normal epimorphism, we conclude that $w$ is an isomorphism.

Assuming that condition 2.1(b) holds, we can say that the conditions/axioms used here are much weaker than those used by G. Janelidze, L. Márki, and W. Tholen [**7**]. However, various arguments from [**7**], used there in the proof of the equivalence of the so-called old and new axioms, can be extended to our context to obtain various reformulations of our conditions. Some of them are given in this section.

**Condition 2.2.** *(a) Every morphism in* $\mathbf{E}$ *is a regular epimorphism;*

*(b) If* $f \in \mathbf{E}$ *then* $\mathrm{coker}(\ker(f)) \in \mathbf{E}$*;*

*(c) ("Relative Hofmann's axiom") If if in a commutative diagram*

$$\begin{tikzcd} A \arrow[r, "f"] \arrow[d, "w"'] & B \arrow[d, "v"] \\ A' \arrow[r, "f'"'] & B' \end{tikzcd}$$

*$f$ and $f'$ are in* $\mathbf{E}$*, $w$ is a monomorphism, $v$ is normal monomorphism, and* $\ker(f') \leqslant w$*, then $w$ is a normal monomorphism;*

*(d) If in a commutative diagram*

$$\begin{tikzcd} A \arrow[rd, "e_1"] \arrow[dd, "f"'] & \\ & C \\ B \arrow[ru, "e_2"'] & \end{tikzcd}$$

*the morphisms $e_1$ and $e_2$ are in* $\mathbf{E}$ *and* $\mathrm{Ker}(e_1) = \mathrm{Ker}(e_2)$*, then there exists a factorization $f = me$, in which $m$ is a monomorphism and $e$ is in* $\mathbf{E}$*.*

**Theorem 2.3.** *(i) Condition 2.1(b) implies conditions 2.2(a) and 2.2(b);*

*(ii) Conditions 2.1(a), 2.1(c), 2.2(a) and 2.2(b) imply condition 2.1(b).*

*Proof.* (i) is obvious.

(ii): Let $f : A \to B$ be a regular epimorphism in $\mathbf{E}$, and let $k = \ker(f)$ and $q = \mathrm{coker}(k)$; then condition 2.2(b) implies that $q$ also is in $\mathbf{E}$. To prove that $f$ is a normal epimorphism, it is sufficient to show that the canonical morphism $h : \mathrm{Coker}(k) \to B$ is an isomorphism. For, consider the commutative diagram

$$\begin{tikzcd} S \arrow[r, shift left, "s_1"] \arrow[r, shift right, "s_2"'] \arrow[d, "\bar{h}"'] & A \arrow[r, "q"] \arrow[rrd, "f"'] & \mathrm{Coker}(k) \arrow[d, "h"] \\ R \arrow[ru, bend left, shift left, "r_1"] \arrow[ru, bend left, shift right, "r_2"'] & & B \end{tikzcd}$$

in which:

- $(r_1, r_2)$ is the kernel pair of $f$ and $(s_1, s_2)$ is the kernel pair of $q$; since the class $\mathbf{E}$ is pullback stable (condition 2.1(a)), the morphisms $r_1$,$r_2$,$s_1$, and $s_2$ are in $\mathbf{E}$.
- $\bar{h} : S \to R$ is induced by $h$.

Since there are canonical isomorphisms

$$\mathrm{Ker}(s_1) \approx \mathrm{Ker}(q) \approx \mathrm{Ker}(f) \approx \mathrm{Ker}(r_1),$$

we can apply condition 2.1(c) to the diagram

$$\begin{tikzcd}
\mathrm{Ker}(s_1) \arrow[r] \arrow[d, "\approx"'] & S \arrow[r, "s_1"] \arrow[d, "\bar{h}"'] & A \arrow[d, equal] \\
\mathrm{Ker}(r_1) \arrow[r] & R \arrow[r, "r_1"'] & A
\end{tikzcd}$$

This makes $\bar{h}$ an isomorphism; since $f$ and $q$ are regular epimorphisms, the latter implies that $h$ also is an isomorphism. ☐

**Theorem 2.4.** *(i) Conditions 2.1(a) and 2.1(c) imply condition 2.2(c);*

*(ii) Condition 2.1(c) implies condition 2.2(d);*

*(iii) Conditions 2.1(b), 2.2(c), and 2.2(d) imply condition 2.1(c).*

*Proof.* (i): According to the assumptions of 2.2(c), consider the commutative diagram

$$\begin{tikzcd}
K \arrow[r, "k"] \arrow[d, equal] & A \arrow[r, "f"] \arrow[d, "w"'] & B \arrow[d, "v"] \\
K \arrow[r, "k'"'] & A' \arrow[r, "f'"'] & B'
\end{tikzcd}$$

in which $f$ and $f'$ are in $\mathbf{E}$, $k' = \ker(f')$, $k$ is a morphism with $wk = k'$, $w$ is a monomorphism, and $v$ is a normal monomorphism. It is easy to see that $k$ is in fact the kernel of $f$, and therefore condition 2.1(c) can be applied to the diagram

$$\begin{tikzcd}
K \arrow[r, "k"] \arrow[d, equal] & A \arrow[r, "f"] \arrow[d, "\langle w,f \rangle"'] & B \arrow[d, equal] \\
K \arrow[r, "h'"'] & A' \times_{B'} B \arrow[r, "\pi_2"'] & B
\end{tikzcd}$$

where the projection $\pi_2$, being the pullback of $f'$ along $v$, is in $\mathbf{E}$ by condition 2.1(a). It follows that $\langle w, f \rangle$ is an isomorphism, and so $w$ is the pullback of $v$ along $f'$. Since normal monomorphisms are pullback stable, we conclude that $w$ is a normal monomorphism, as desired.

(ii): Since $\mathbf{E}$ contains all isomorphisms, condition 2.2(d) follows directly from condition 2.1(c).

(iii): We have to show that if in a commutative diagram

$$\begin{tikzcd} K \arrow[r, "k"] \arrow[d, equal] & A \arrow[r, "f"] \arrow[d, "w"'] & B \arrow[d, equal] \\ K \arrow[r, "k'"'] & A' \arrow[r, "f'"'] & B \end{tikzcd}$$

$f$ and $f'$ are in $\mathbf{E}$, and $k$ and $k'$ are their kernels respectively, then $w$ is an isomorphism.

It is a well known fact, that in the situation above $w$ has zero kernel and zero cokernel. By conditions 2.2(d) and 2.1(b), we have a factorization $w = me$ in which $m$ is a monomorphism and $e$ is a normal epimorphism in $\mathbf{E}$. Moreover, since $w$ has zero kernel, $e$ is an isomorphism. Therefore $w$ is a monomorphism, and applying condition 2.2(c) to the diagram above, we conclude that $w$ is a normal monomorphism. Since $w$ has zero cokernel, this implies that $w$ is an isomorphism, as desired. ☐

Combining these two theorems, we obtain:

**Corollary 2.5.** *The following conditions are equivalent:*

*(i) A pair* $(\mathbf{C}, \mathbf{E})$ *is a relative weakly homological category;*

*(ii) A pair* $(\mathbf{C}, \mathbf{E})$ *satisfies conditions 2.1(a), 2.1(c), 2.1(d), 2.1(e), 2.2(a), and 2.2(b);*

*(iii) A pair* $(\mathbf{C}, \mathbf{E})$ *satisfies conditions 2.1(a), 2.1(b), 2.1(d), 2.1(e), 2.2(c), and 2.2(d).*

*Proof.* The implications (i)⇒(ii), (ii)⇒(i), (i)⇒(iii), and (iii)⇒(i) follow from 2.3(i), 2.3(ii), 2.4(i)-(ii), and 2.4(iii) respectively. ☐

## 3. Examples: protomodular and homological categories

**Proposition 3.1.** *The following conditions are equivalent:*

*(i) A pair* $(\mathbf{C}, \mathbf{E})$ *in which* $\mathbf{E}$ *is the class of all split epimorphisms in* $\mathbf{C}$*, is a relative weakly homological category;*

*(ii)* $\mathbf{C}$ *is a protomodular category in the sense of D. Bourn [4].*

*Proof.* The implication (i)⇒(ii) follows directly from the definitions.

(ii)⇒(i): The only condition that requires a verification here is 2.1(b); however it holds by Proposition 3.1.23 of [**2**], which asserts that in a pointed protomodular category with finite limits, a morphism $f$ is a regular epimorphism if and only if $f = \mathrm{coker}(\ker(f))$. ☐

**Proposition 3.2.** *If* $\mathbf{C}$ *has coequalizers of kernel pairs and* $\mathbf{E}$ *is the class of all regular epimorphisms in* $\mathbf{C}$*, then the following conditions are equivalent:*

*(i)* $(\mathbf{C}, \mathbf{E})$ *is a relative weakly homological category;*

*(ii)* $(\mathbf{C}, \mathbf{E})$ *is a relative homological category;*

*(iii)* **C** *is a homological category in the sense of F. Borceux and D. Bourn [**2**].*

*Proof.* (i)⇒(iii): As follows from (i), the class of all regular epimorphisms in **C** is pullback stable. Therefore, since **C** has kernel pairs and their coequalizers, it admits (regular epi, mono)-factorization system. Furthermore, using the same arguments as in [**5**], one can show that in this situation, protomodularity is equivalent to the **E**-short five lemma. It follows that **C** is a homological category.

(iii)⇒(ii): Let **C** be a homological category and **E** be the class of all regular epimorphisms in **C**. Then all properties we need for $(\mathbf{C}, \mathbf{E})$ to be a relative homological category, are proved in [**2**].

Since the implication (ii)⇒(i) is trivial, this completes the proof. □

**Example 3.3.** *Let* $(\mathbf{C}, \mathbf{E})$ *be a relative weakly homological category and let* $(\mathbf{C}', \mathbf{E}')$ *be a pair, in which* $\mathbf{C}'$ *is a category with finite limits and* $\mathbf{E}'$ *is a class of morphisms in* $\mathbf{C}'$ *satisfying conditions 2.1(a), 2.1(d), and 2.1(e). If functor* $F : \mathbf{C} \to \mathbf{C}'$ *preserves finite limits, then the pair* $(\mathbf{C}, \mathbf{E} \cap F^{-1}(\mathbf{E}'))$*, in which* $F^{-1}(\mathbf{E}')$ *is the class of all morphisms* $e$ *from* **E** *for which* $F(e)$ *is in* $\mathbf{E}'$*, is a relative weakly homological category. In particular we could take* $\mathbf{C}'$ *to be an arbitrary category with finite limits and* $\mathbf{E}' =$ **SplitEpi** *to be the class of all split epimorphisms in* $\mathbf{C}'$*. According to the existing literature (see e.g. [**11**]), an important example is provided by the forgetful functor* $F$ *from the homological category* **C** *of topological groups to the category* $\mathbf{C}'$ *of topological spaces; the class* $F^{-1}$(**SplitEpi**) *and the corresponding concept of exactness play a significant role in the cohomology theory of topological groups. This also applies to the classical case of profinite groups, where, however,* $F^{-1}$(**SplitEpi**) *coincides with the class of all normal epimorphisms, as shown in Section I.1.2 of [**10**]; another such result is used in [**9**]. The results of [**6**] also suggest considering the forgetful functor from the category of topological groups to the category of groups. On the other hand one can replace topological groups with more general, so-called protomodular (=semi-abelian), topological algebras, which form a homological category due to a result of F. Borceux and M. M. Clementino [**3**].*

Let us also mention the following "trivial" examples:

**Example 3.4.** *If* **C** *is an abelian category, and* **E** *is the proper class of epimorphisms in* **C** *in the sense of relative homological algebra (see e.g. Chapter IX in [**8**]) then* $(\mathbf{C}, \mathbf{E})$ *is a relative weakly homological category.*

**Example 3.5.** *A pair* $(\mathbf{C}, \mathbf{E})$*, in which* **E** *is the class of all isomorphisms in* **C***, always is a relative homological category.*

**Example 3.6.** *A pair* $(\mathbf{C}, \mathbf{E})$*, in which* **E** *is the class of all morphisms in* **C***, is a relative homological category if and only if* **C** *is a trivial category.*

## 4. Remarks on homological lemmas

**Definition 4.1.** *Let* $(\mathbf{C}, \mathbf{E})$ *be a relative homological category. A sequence of morphisms*

$$\cdots \longrightarrow A_{i-1} \xrightarrow{f_{i-1}} A_i \xrightarrow{f_i} A_{i+1} \longrightarrow \cdots$$

*is said to be:*

*(i)* $\mathbf{E}$*-exact at* $A_i$*, if the morphism* $f_{i-1}$ *admits a factorization* $f_{i-1} = me$*, in which* $e \in \mathbf{E}$ *and* $m = \ker(f_i)$*;*

*(ii) an* $\mathbf{E}$*-exact sequence, if it is* $\mathbf{E}$*-exact at* $A_i$ *for each* $i$ *(unless the sequence either begins with* $A_i$ *or ends with* $A_i$*).*

As easily follows from the definition, the sequence

$$0 \longrightarrow A \xrightarrow{f} B \xrightarrow{g} C \longrightarrow 0$$

is $\mathbf{E}$-exact, if and only if $f = \ker(g)$ and $g \in \mathbf{E}$. Having this notion of $\mathbf{E}$-exact sequences, we may consider the relative cases of various homological lemmas from [**1**] and [**2**].

**Relative snake lemma.** Using the same arguments as in the proof of the Theorem 4.4.2 of [**2**], we can prove the following

**Lemma 4.2 (Relative snake lemma).** *Let* $(\mathbf{C}, \mathbf{E})$ *be a relative homological category. Consider the commutative diagram*

$$\begin{tikzcd}
 & 0 \arrow[d] & 0 \arrow[d] & 0 \arrow[d] & \\
 & \mathrm{Ker}(u) \arrow[r] \arrow[d] & \mathrm{Ker}(v) \arrow[r] \arrow[d] & \mathrm{Ker}(w) \arrow[d] & \\
 & A \arrow[r, "f"] \arrow[d, "u"] & B \arrow[r, "g"] \arrow[d, "v"] & C \arrow[r] \arrow[d, "w"] & 0 \\
0 \arrow[r] & A' \arrow[r, "f'"] \arrow[d] & B' \arrow[r, "g'"] \arrow[d] & C' \arrow[d] & \\
 & \mathrm{Coker}(u) \arrow[r] \arrow[d] & \mathrm{Coker}(v) \arrow[r] \arrow[d] & \mathrm{Coker}(w) \arrow[d] & \\
 & 0 & 0 & 0 &
\end{tikzcd}$$

*in which all columns, and the second and the third rows are* $\mathbf{E}$*-exact sequences. Suppose* $g' \in \mathbf{E}$*, or, more generally, the following conditions hold:*

*(a)* *If* ${g_1}' = \operatorname{coker}(f')$ *and* ${g_2}'$ *is the unique morphism with* ${g_2}'{g_1}' = g'$*, then* ${g_1}'$ *is in* $\mathbf{E}$ *and* ${g_2}'$ *is a monomorphism;*

*(b)* $(\operatorname{coker}(w))g_2'$ *is in* $\mathbf{E}$*.*

*Then there exists a natural morphism* $d : \operatorname{Ker}(w) \to \operatorname{Coker}(u)$*, such that the sequence*

$$\operatorname{Ker}(u) \longrightarrow \operatorname{Ker}(v) \longrightarrow \operatorname{Ker}(w) \xrightarrow{d} \operatorname{Coker}(u) \longrightarrow \operatorname{Coker}(v) \longrightarrow \operatorname{Coker}(w)$$

*is* $\mathbf{E}$*-exact.*

Note that the relative snake lemma can be proved in a relative weakly homological category $(\mathbf{C}, \mathbf{E})$ under some additional conditions. These additional conditions, however, easily follow from 2.1(f) and 2.1(g) when $(\mathbf{C}, \mathbf{E})$ is a relative homological category. Being more precise, to construct the morphism $d$ we only need condition 4.2(a), but to prove the $\mathbf{E}$-exactness of the sequence above, we need the following:

- Condition 4.2(b);
- The morphisms $(\operatorname{coker}(v))f'$ and $(\operatorname{coker}(w))g_2'$ admit a (normal epi in $\mathbf{E}$, mono)-factorization;
- If $(B \times_C (\operatorname{Ker}(w)), \pi_1, \pi_2)$ is a pullback of $g$ and $\ker(w)$, $(A' \times_{(\operatorname{Coker}(u))} (\operatorname{Ker}(w)), {\pi_1}', {\pi_2}')$ is a pullback of $\operatorname{coker}(u)$ and $d$, and if $\varphi : B \times_C (\operatorname{Ker}(w)) \to A'$ is induced by $f'$ and $v\pi_1 : B \times_C (\operatorname{Ker}(w)) \to B'$, then the morphism $\langle \varphi, \pi_2 \rangle : B \times_C (\operatorname{Ker}(w)) \to A' \times_{(\operatorname{Coker}(u))} (\operatorname{Ker}(w))$ is in $\mathbf{E}$.

**Relative 3×3-lemma.** Using the same arguments as in the proof of the Theorem 4.5 of [**1**], we can prove the following

**Lemma 4.3 (Relative 3×3-lemma).** *Let* $(\mathbf{C}, \mathbf{E})$ *be a relative homological category. If in a commutative diagram*

$$\begin{tikzcd}
 & 0 \arrow[d] & 0 \arrow[d] & 0 \arrow[d] & \\
0 \arrow[r] & A \arrow[r, "f"] \arrow[d, "u"'] & B \arrow[r, "g"] \arrow[d, "v"'] & C \arrow[r] \arrow[d, "w"] & 0 \\
0 \arrow[r] & A' \arrow[r, "f'"'] \arrow[d, "u'"'] & B' \arrow[r, "g'"'] \arrow[d, "v'"'] & C' \arrow[r] \arrow[d, "w'"] & 0 \\
0 \arrow[r] & A'' \arrow[r, "f''"'] \arrow[d] & B'' \arrow[r, "g''"'] \arrow[d] & C'' \arrow[r] \arrow[d] & 0 \\
 & 0 & 0 & 0 &
\end{tikzcd}$$

*the three columns and the middle row are* $\mathbf{E}$*-exact sequences, then the first row is an* $\mathbf{E}$*-exact sequence if and only if the last row is an* $\mathbf{E}$*-exact sequence.*

More generally, the relative version of the 3×3-lemma can be proved in a relative weakly homological category, if the following condition holds:

- The morphism $\langle v', g' \rangle : B' \to B'' \times_{C''} C'$ (where $(B'' \times_{C''} C', \pi_1, \pi_2)$ is a pullback of $g''$ and $w'$) is in $\mathbf{E}$.

It is easy to show that if $(\mathbf{C}, \mathbf{E})$ is a relative homological category, then this condition follows from Definition 2.1(g).

**Remark 4.4.** *If* $\mathbf{E}$ *is the class of all regular epimorphisms in* $\mathbf{C}$*, then, by Proposition 3.2,* $\mathbf{C}$ *is a homological category and all the extra conditions needed for the proofs of the relative homological lemmas are satisfied. In this case, the relative snake lemma coincides with its "absolute version" proved in [**2**], and the relative 3×3-lemma coincides with its "absolute version" proved in [**1**].*

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