Journal of Homotopy and Related Structures, vol. 2(2), 2007, pp.7–10

AN UNDERGRADUATE'S MEMORY OF SAUNDERS MAC LANE

WOJCIECH KOMORNICKI

(*communicated by Ronald Brown*)

Abstract

Saunders Mac Lane was not only a leader in the mathematics community but also an inspiration to working mathematicians, graduate students and also undergraduates. In teaching generations of undergraduate students he inspired and laid down a standard for future generations of teachers. He taught many that the beauty of mathematics can and should be communicated to all.

Much has been written about the role of Saunders Mac Lane in the life of American Mathematics, his leadership in professional organizations, his influence on the direction of mathematical research, his mentoring of graduate students. I would like to write about a very different role that Mac Lane played, that of his influence on undergraduates who approached mathematics with enthusiasm for the subject and hope that their early success in mathematics courses might indeed flower. During his years at Chicago, Mac Lane taught a number of undergraduate courses and touched both those who took his courses and who had the good fortune of meeting him at departmental teas, in his office or his apartment. To set things in context, please allow me to talk a little about how I came to meet Saunders Mac Lane.

I was introduced to abstract mathematics early in high school. In 9^{th} grade my mathematics teacher had the audacity to give the class the definition of the reals as a totally ordered Archimedean field. He did not use those words, but he did hand out two pages of the axioms for a field and ending with the Archimedean property. Using these we proved many properties. It was my first experience that things such as "the product of any number and zero is zero" could be proved. I thought it was the greatest thing since sliced bread. This same teacher was instrumental in pushing me to attend a summer NSF program in which I had my first full introduction to group theory. There I was told that to go on in algebra I should get Birkhoff-Mac Lane's book. When I got to Chicago and attended during orientation week my first Mathematics Department Tea in Eckhart Hall, Saunders Mac Lane came up to me, introduced himself and spent a few moments welcoming me to mathematics at Chicago. I was in awe. To a shy, naive, hopeful student of mathematics here was a legend, a towering figure in world mathematics talking to me. I was to find through my next four years of undergraduate work, and then later during graduate school

Received August 1, 2006; published on December 27, 2007.
2000 Mathematics Subject Classification: 01-00
Key words and phrases: Mac Lane

and later that Mac Lane was always approachable and genuinely interested in what students were doing.

As a first year student, I did not have any more personal contact with Mac Lane. However, I heard a lot about him. At that time, 1967-68, he was teaching from the yet to be published Mac Lane-Birkhoff *Algebra* and an upperclassman in my dorm was taking the course. Eugene would tell me about objects and arrows and universal diagrams. This was nothing like any other mathematics course he had taken. I learned more about who Mac Lane was. But I also heard of rumors that he might leave Chicago. I never did find out whether the rumors were true but did decide to try to take his algebra course the following year even though I did not have all the prerequisites I obtained permission to enroll in the algebra course. Eugene had been right; it was nothing like any mathematics course I had ever taken. Though teaching from his own book, it seemed that Mac Lane would go out of his way to present the material differently than what was in the text. Sometimes he would give more than one proof of the same theorem. He made sure that we were aware of this, explaining that the more ways we understood something the better our understanding. Though I once again saw the same definition of a group that I remembered from my NSF program, I heard that a group was a category with one object in which all the arrows were invertible. The product of two groups was a universal object. I remember struggling with these new concepts especially since Mac Lane's style was to give us the big picture leaving most, but not all, of the details to the book and to the problems he assigned. However, the understanding of the mathematics was paramount in Mac Lane's presentations. If he thought that someone did not understand a proof, he would provide an alternative proof. And there was never any hint that a concept or a proof was too complex for someone to understand. Another aspect of his teaching that struck me was that he never came to class with notes. He would every once in a while pull out an index card on which I assume he had the subject matter of the day's class, look at it, put it back in his shirt or jacket pocket and continue the lecture.

Though his lectures were meticulous in presenting the global picture of what we were learning he took great care to answer questions. No question was too small and he never blew off a question nor showed any irritation by student asking questions. I had the annoying habit to ask questions at times before he had finished a sentence - at least I am sure that is how it seemed. In my fourth year I was taking a course from Mac Lane and blurted out a question. Mac Lane stopped the lecture, took off his glasses and turned to the class telling my fellow students that I had been his "chief heckler" for the past four years. After the aside, he proceeded to address my question.

However, he did make sure that we did understand that he was not going to do our thinking for us. In an algebra course he handed out a list of 20 topics/problems from which we were to pick one and then present a talk/solution in his office. I chose the classical Greek problem of the impossibility of trisecting an angle by compass and straight edge. We had already covered field extensions in the course so this was not beyond the scope of what we should be able to do. I prepared my talk and started confidently. Unfortunately, I made the fatal mistake at one point of stating “and it is obvious that ...” Mac Lane, as I was finishing the sentence, said that he did not see that it was obvious. I tried to explain but that did not satisfy him. Even with almost 40 years of hindsight it seemed that he needled me for an interminably long time. (As I think back on it, “how could I have been that stupid not to say the right words.”) Finally, I was allowed to finish the talk. On leaving his office, feeling totally beaten into the ground, Mac Lane stopped me and out of the earshot of other students told me that he expected better from me. He also in those few words somehow made me feel better. Though he could be extremely critical of one’s mathematical work, the criticism was never personal. I never felt that he was putting me down. His commentary was always indicative of where there were errors in the mathematics in order to improve that mathematics. The memory of that talk has stayed with me and I have tried to remember never to leave the smallest detail unexamined. As a teacher, I now strive to encourage students to work to the maximum of their potential but never to belittle them in their failures. The feeling of students was that he considered us, undergraduate students, as only different from senior mathematicians in experience and not in what we would eventually accomplish.

Mac Lane’s personality often came out in his courses. When introducing noetherian rings, he referred to Emmy Noether as Emmy and regaled us with stories of his days in Germany. However, the stories were always focused on the mentioned mathematicians and not about his encounters with these mathematicians. He talked about his days at Gottingen as a graduate student. I recall the day when he came to class and the blackboard had not been cleaned. Mac Lane looked at the board and turned to us, made some comment about it and walked out of the room. He came back with a pail of water and a sponge with which he proceeded to wash the board. As the board was drying, he told us of how as a graduate student he would come in early and wash the board for I do not remember whom. He looked at us expecting that at least one of us would understand that this was expected behavior. None of us took him up on the opportunity to wash the board before class and so Mac Lane, for the rest of the quarter came to class with a bucket of water and a sponge and washed the board.

Saunders Mac Lane’s personality came out at other times. Every once in while he would say something to us in German. Upon seeing blank faces he would wonder out loud how is it possible that we did not understand German. His plaid pants and wild ties were legendary. The trousers were an aspect of his Scottish background. However, the ties. I remember one with small round mirrors in increasing sizes from top to bottom. It reflected the lights in the class extremely well and I probably paid more attention to that tie than to what was being written on the board. Students would at times approach Mac Lane and ask about his background, his days in Germany, etc. He was always willing to chat with us. One day, several of the undergraduates from his class were greatly honored by an invitation to come to his apartment for dinner. There were a few other mathematicians from the department at the dinner but the conversation was much more about everyday things and people rather than about mathematics. I remember Mac Lane telling us that his daughter was living in Scotland. One of us asked why she was there. Mac Lane gave us that look of astonishment he used when conveying that something so obvious should not need explanation: “Why, for the best reason of all. Her husband is there.”

Even when I was not taking a class with Mac Lane, I felt that I could knock on his door and get advice and direction. I delayed taking multivariable calculus until my third year. The course had the title “Honors Calculus in R^n” and the first day the instructor made it known that since this was an honors class, we would be stretched a bit further than we may expect. By the third quarter we were covering differential forms and since the TA for the course was writing his thesis in algebraic topology, we were introduced to de Rham cohomology. This was not calculus. Looking back on this, I am embarrassed that I did not stop to think about the calculus but concentrated on the *cohomology*. Again, as in Mac Lane’s class, we were given a choice of various topics to investigate and present in the instructor’s office. I chose to prove that cohomology is homotopy invariant. Since the only type of cohomology we had seen was that presented by the TA, de Rham’s name was not mentioned in the phrasing of the problem. I made the disastrous mistake of trying to find a solution to my problem by reading Spanier’s *Algebraic Topology* instead of sitting down and thinking about what was presented in class. By the time I got to acyclic models I was completely confused and decided to seek Mac Lane’s help. Mac Lane greeted me at his office door and listened to my story and my confusion. He told me that acyclic models were *the* way of proving the homotopy invariance of cohomology and that the presentation was “in a book by Mac Lane.” I found it curious that he referred to himself in the third person but even more so that he went to his bookshelf, reached for his *Homology* and loaned it to me.

After I left Chicago and started graduate work, I would see Mac Lane at various conferences. At times he would notice me sitting somewhere pondering the last talk or working on some problem. He would come over, sit down next to me and inquire how my studies were going. I am not sure that he always remembered my name but he remembered that I had been in his classes and that I benefited from his guidance. The last time I saw him was in the 90’s on a Saturday in Eckhart Hall at the University of Chicago. I was attending a meeting and during a break was perusing the bulletin board. Mac Lane came walking down the hall with a cane in one hand and a patch over an eye.

He stopped, said hello and chatted with me about my work. This was so typical of Saunders Mac Lane to show interest in another. I hope that I can convey to my students, as Mac Lane did for so many undergraduates, the beauty of mathematics and the confidence that mathematics can be “done.”

This article may be accessed via WWW at `http://jhrs.rmi.acnet.ge`

Wojciech Komornicki
`wnk@hamline.edu`

Department of Mathematics
Hamline University
St Paul, MN 55113
USA